Controlling the Population

12.1

Adding and Subtracting Polynomials

LEARNING GOALS

In this lesson, you will:

- Recognize polynomial expressions.
- Identify monomials, binomials, and trinomials.
- Identify the degree of a term and the degree of a polynomial.
- Write polynomial expressions in standard form.
- Add and subtract polynomial expressions.
- Graph polynomial functions and understand the connection between the graph of the solution and the algebraic solution.

KEY TERMS

- polynomial
- term
- coefficient
- monomial
- binomial
- trinomial
- degree of a term
- degree of a polynomial

There are all kinds of talking birds. The common crow is able to repeat a few words, while a bird called a Bedgerigar (or common parakeet) can have a vocabulary of thousands of words. A bird of this type named Puck was found in 1995 to have a vocabulary of 1728 words.

African Grey Parrots are also remarkable, not only for their knowledge of words, but also for their other mental abilities. In 2005, an African Grey Parrot named Alex was reported to have understood the concept of zero!

© 2012 Carnegie Learning

PROBLEM 1 Many Terms

Previously, you worked with a number of expressions in the form $ax + b$ and $ax^2 + bx + c$. Each of these is also part of a larger group of expressions known as *polynomials*.

A **polynomial** is a mathematical expression involving the sum of powers in one or more variables multiplied by coefficients. A polynomial in one variable is the sum of terms of the form ax^k, where a is any real number and k is a non-negative integer. In general, a polynomial is of the form $a_1x^k + a_2x^{k-1} + \ldots a_nx^0$. Within a polynomial, each product is a **term**, and the number being multiplied by a power is a **coefficient.**

The polynomial $m^3 + 8m^2 - 10m + 5$ has four terms.
Each term is written in the form ax^k.

First term: m^3

coefficient: $a = 1$

variable: m

power: m^3

exponent: $k = 3$

1. Write each term from the worked example and then identify the coefficient, power, and exponent. The first term has already been completed for you.

	1st	2nd	3rd	4th
Term	m^3			
Coefficient	1			
Power	m^3			
Exponent	3			

2. Analyze each polynomial. Identify the terms and coefficients in each.

   a. $-2x^2 + 100x$

   b. $x^2 + 4x + 3$

   c. $4m^3 - 2m^2 + 5$

© 2012 Carnegie Learning

Polynomials are named according to the number of terms they have. Polynomials with only one term are **monomials.** Polynomials with exactly two terms are **binomials.** Polynomials with exactly three terms are **trinomials.**

The **degree of a term** in a polynomial is the exponent of the term. The greatest exponent in a polynomial determines the **degree of the polynomial.** In the polynomial $4x + 3$, the greatest exponent is 1, so the degree of the polynomial is 1.

3. Khalil says that $3x^{-2} + 4x - 1$ is a polynomial with a degree of 1 because 1 is the greatest exponent and it is a trinomial because it has 3 terms. Jazmin disagrees and says that this is not a polynomial at all because the power on the first term is not a whole number. Who is correct? Explain your reasoning.

4. Describe why each expression is not a polynomial.

a. $\frac{4}{x}$

12

b. $\sqrt{x}$

5. Cut out each polynomial.

Identify the degree of each polynomial and then analyze and sort according to the number of terms of the polynomial. Finally, glue the sorted polynomials in the appropriate column of the table.

$4x - 6x^2$ Degree:	$125p$ Degree:	$\frac{4}{5}r^3 + \frac{2}{5}r - 1$ Degree:
$-\frac{2}{3}$ Degree:	$y^2 - 4y + 10$ Degree:	$5 - 7h$ Degree:
$-3 + 7n + n^2$ Degree:	-6 Degree:	$-13s + 6$ Degree:
$12.5t^3$ Degree:	$78j^3 - 3j$ Degree:	$25 - 18m^2$ Degree:

12

Monomial	Binomial	Trinomial
Degree:	Degree:	Degree:
Degree:	Degree:	Degree:
Degree:	Degree:	Degree:
Degree:	Degree:	
	Degree:	

12

A polynomial is written in standard form when the terms are in descending order, starting with the term with the greatest degree and ending with the term with the least degree.

6. Analyze the polynomials you just sorted. Identify the polynomials not written in standard form and write them in standard form.

PROBLEM 2 Something to Squawk About

You are playing a new virtual reality game called "Species." You are an environmental scientist who is responsible for tracking two species of endangered parrots, the Orange-bellied Parrot and the Yellow-headed Parrot. Suppose the Orange-bellied Parrots' population can be modeled by the function

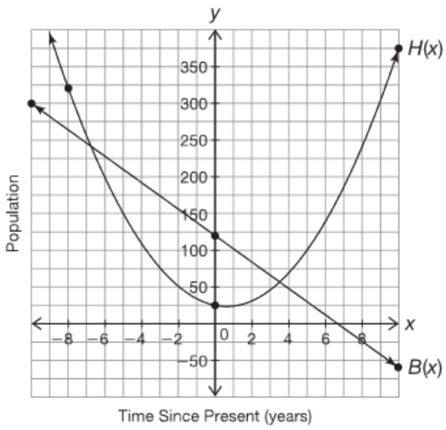
$$B(x) = -18x + 120,$$

where x represents the number of years since the current year. Then suppose that the population of the Yellow-headed Parrot can be modeled by the function

$$H(x) = 4x^2 - 5x + 25.$$

The graphs of the two polynomial functions are shown.

Your new task in the game is to determine the total number of these endangered parrots over a six-year span. You can calculate the total population of parrots using the two graphed functions.



1. Use the graphs of $B(x)$ and $H(x)$ to determine the function, $T(x)$, to represent the total population of parrots.
   a. Write $T(x)$ in terms of $B(x)$ and $H(x)$.

   b. Predict the shape of the graph of $T(x)$.

12

   c. Sketch a graph of $T(x)$ on the coordinate plane shown. First choose any 5 x-values and add their corresponding y-values to create a new point on the graph of $T(x)$. Then connect the points with a smooth curve.

   d. Did your sketch match your prediction in part (b)? Describe the function family $T(x)$ belongs to.

© 2012 Carnegie Learning

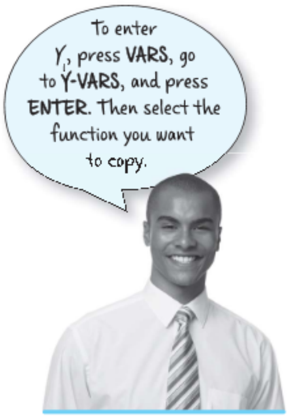
2. Use a graphing calculator to check your sketch by graphing $B(x)$ and $H(x)$ and then the sum of the two functions.

a. In Y_1, enter the function for $B(x)$.

b. In Y_2, enter the function for $H(x)$.

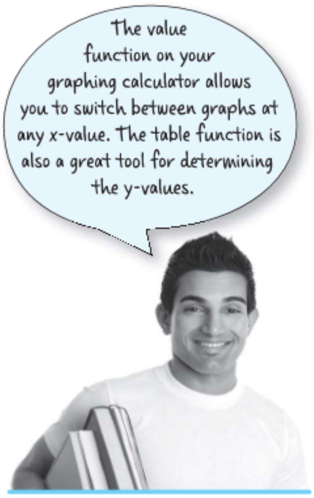
c. In Y_3, enter the sum of the functions $B(x)$ and $H(x)$.

(Since $Y_1 = B(x)$ and $Y_2 = H(x)$, you can enter $Y_1 + Y_2$ in Y_3 to represent the sum of $B(x)$ and $H(x)$.)



3. Complete the table.

Time Since Present (years)	Number of Orange-bellied Parrots	Number of Yellow-headed Parrots	Total Number of Parrots
−3			
−2			
−1			
0			
1			
2			
3			



12

4. How did you calculate the total number of parrots?

5. Write a function, $T(x)$, in terms of x that can be used to calculate the total number of parrots at any time.

6. Analyze the function you wrote in Question 5.

a. Identify the like terms of the polynomial functions.

b. Use the Associative Property to group the like terms together.

c. Combine like terms to simplify the expression.

d. Graph the function you wrote in part (c) to verify that it matches the graph of the sum of the two functions in Y_3.

7. Use your new polynomial function to confirm that your solution in the table for each time is correct.

a. 3 years ago

b. currently

c. 3 years from now

12

8. Zoe says that using *T*(*x*) will not work for any time after 6 years from now because by that point the Orange-bellied Parrot will be extinct. Is Zoe's statement correct? Why or why not?

© 2012 Carnegie Learning

Throughout the game "Species," you must always keep track of the difference between the populations of each type of species. If the difference gets to be too great, you lose the game.

9. Use the graphs of $B(x)$ and $H(x)$ to determine the function, $D(x)$, to represent the difference between the populations of each type of species.

a. Write $D(x)$ in terms of $B(x)$ and $H(x)$.

b. Predict the shape of the graph of $D(x)$.

c. Sketch a graph of $D(x)$ on the coordinate plane in Question 1. Choose any 5 x-values and subtract their corresponding y-values to create a new point to form the graph of $D(x)$. Then connect the points with a smooth curve.

d. Did your sketch match your prediction in part (b)? Describe the function family $D(x)$ belongs to.

10. Use a graphing calculator to check your sketch by graphing $B(x)$ and $H(x)$ and then the difference of the two functions.

a. For Y_1, enter the function for $B(x)$.

b. For Y_2, enter the function for $H(x)$.

c. For Y_3, enter the difference of the functions $B(x)$ and $H(x)$.

(Since $Y_1 = B(x)$ and $Y_2 = H(x)$, you can enter $Y_1 - Y_2$ in Y_3 to represent $B(x) - H(x)$.)

11. Consider the original functions, $B(x) = -18x + 120$ and $H(x) = 4x^2 - 5x + 25$.

a. Write a new function, $D(x)$, in terms of x to represent the difference between the population of Orange-bellied Parrots and the population of Yellow-headed Parrots.

b. Because you are subtracting one function from another function, you must subtract each term of the second function from the first function. To do this, use the Distributive Property to distribute the negative sign to each term in the second function.

c. Now, combine the like terms to simplify the expression.

d. Graph the function you wrote in part (c) to verify that it matches the graph of the difference of the two functions in Y_3.

12. For each time given, use the table in Question 2 to determine the difference between the population of Orange-bellied Parrots and the population of Yellow-headed Parrots. Then, use the function you wrote in Question 8, part (c) to verify your solutions.

a. 3 years ago

b. currently

c. 3 years from now

12

© 2012 Carnegie Learning

13. Eric uses his function function, $D(x) = -4x^2 - 13x + 95$, to determine that the difference between the number of Orange-bellied Parrots and the number of Yellow-headed Parrots 7 years from now will be 192. Is Eric correct or incorrect? If he is correct, explain to him what his answer means in terms of the problem situation. If he is incorrect, explain where he made his error and how to correct it.

$D(x) = -4(7)^2 - 13(7) + 95$
$= -196 - 91 + 95$
$D(x) = -192$

PROBLEM 3 Just the Math

1. Analyze the work. Determine the error and make the necessary corrections.

a.

Marco

$3x^2 + 5x^2 = 8x^4$

b.

Kamiah

$2x - (4x + 5)$
$2x - 4x + 5$
$-2x + 5$

c.

Alexis

$(4x^2 - 2x - 5) + (3x^2 + 7)$
$(4x^2 + 3x^2) - (2x) - (5 + 7)$
$x^2 - 2x - 12$

2. Determine the sum or difference of each. Show your work.

a. $(x^2 - 2x - 3) + (2x + 1)$

b. $(2x^2 + 3x - 4) - (2x^2 + 5x - 6)$

c. $(4x^3 + 5x - 2) + (7x^2 - 8x + 9)$

d. $(9x^4 - 5) - (8x^4 - 2x^3 + x)$

12

© 2012 Carnegie Learning